

NAME:

Math 125 Exam 4

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. Note that you must do at least 10 problems correctly to get 100. Write neatly and legibly in the space provided. **SHOW YOUR WORK!**

Core Problems

1. Let $f(x) = 4\sin(\pi x + \frac{\pi}{2})$.

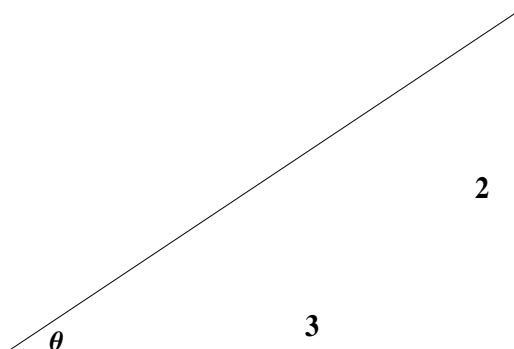
a) What is the amplitude of f ? [1 pt]

b) What is the period of f ? [1 pt]

c) Graph one full cycle (period) of f . Be sure to label the points where f attains its max/min as well as any points where f intersects the x -axis. [8 pts]

2. Find the exact value of $\cos(\frac{\pi}{8})$. Do not use your calculator. [10 pts]

3. Find the exact value of $\cos(2\theta)$ given the diagram below. [10 pts]



4. Find the exact value of each expression, if possible. Otherwise indicate that a solution does not exist. Recall that the domains of the restricted $\sin(x)$, $\cos(x)$, and $\tan(x)$, are respectively $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $[0, \pi]$, and $(-\frac{\pi}{2}, \frac{\pi}{2})$

a) $\sin^{-1}(\sin(\frac{\pi}{7}))$

[2 pts]

b) $\text{Cos}^{-1}(\text{Cos}(\frac{8\pi}{7}))$ [2 pts]

c) $\text{Tan}^{-1}(\text{Tan}(\frac{\pi}{2}))$ [2 pts]

d) $\text{Tan}(\text{Sin}^{-1}(\frac{1}{\sqrt{2}}))$ [2 pts]

e) $\text{Cos}(\text{Cos}^{-1}(\frac{1}{9}))$ [2 pts]

5. Verify the following identity (Hint: Express the left hand side as difference of two squares)

$$\sin^4(\phi) - \cos^4(\phi) = 1 - 2\cos^2(\phi) \quad [10 \text{ pts}]$$

6. Assuming $x > 0$ use a triangle to write $\sec(\sin^{-1} \frac{x}{2})$ algebraically. [10 pts]

7. Hermann Hoth's 4th Panzer army is moving southwards from Orel against Russian defensive fortifications. As part of an elite Pre-Calculus squad, you are in command of a Russian Anti-Tank artillery located in Strong Point A. Suddenly, you see a German Panzer in position C. Your forward observer at position B reports that his line of sight to the Panzer is at an angle of 75° relative to the line segment AB.

The line of sight from the artillery to the Panzer relative to the line segment AB is also 75° .

a) By observing that $75^\circ = 30^\circ + 45^\circ$, find the exact value of $\sin(75^\circ)$ [2 pts]

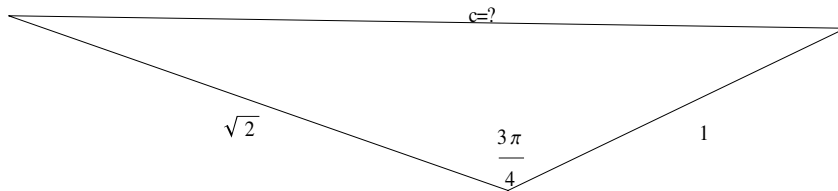
b) If the distance between the forward observer and artillery is 100 meters, what is the distance between the artillery and the Panzer? [6 pts]

Use part (a) to obtain the exact distance. (Remember Comrade, there is no retreat. You must not FAIL!) [2 pts]

8. a) Solve the trigonometric equation $1 - \sin(x) = 2\cos^2(x)$ for all x in the interval $[0, 2\pi)$. [5 pts]

- b) Solve the trigonometric equation $\sin(2x) = \cos(x)$ for all x in the interval $[0, 2\pi)$. [5 pts]

9. Given the following triangle, find the value of c . [10 pts]



10. Given $\sin(\alpha) = \frac{12}{13}$, α in quadrant I and $\tan(\beta) = \frac{3}{4}$, β in quadrant III, Find $\cos(\alpha + \beta)$ [10 pts]

Extra-Credit

11. a) Prove the theorem of Cosines. Namely, prove that if a , b , and c are the sides of a triangle and if θ is an angle between a and b then $c^2 = a^2 + b^2 - 2ab\cos(\theta)$
[10 pts]

- b) Recall that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ where $i = \sqrt{-1}$. Use this identity to derive a formula for $\cos(\alpha + \beta)$ and $\sin(\alpha + \beta)$ in terms of $\cos(\alpha)$, $\cos(\beta)$, $\sin(\alpha)$, and $\sin(\beta)$
[10 pts]